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# **Measurement of Chromatic Dispersion using the Baseband Radio-Frequency Response of a Phase-Modulated Analog Optical Link Employing a Reference Fiber**

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## I EXECUTIVE SUMMARY

The ability to measure chromatic dispersion is essential for all long-haul communications applications – both analog and digital – as well as for ultrashort pulse photonics. Myriad techniques to measure moderate amounts of chromatic dispersion (i.e., dispersion arising from kilometers of optical fiber and the emphasis of this work) have been demonstrated. In this work we demonstrate a new technique for measuring the chromatic dispersion of an optical fiber based using the baseband RF response of a phase-modulated analog optical link in concert with a well-characterized fiber that serves as a dispersion reference. This work:

- Provides a simple method to determine the *magnitude* and *sign* of the chromatic dispersion of an optical fiber;
- Illustrates that optical phase modulation provides increased measurement resolution and immunity to optical modulator bias-drift as compared to baseband methods utilizing optical intensity modulation;
- Gives a simple derivation of the dispersion response of a long analog optical link to both intensity- and phase-modulated signals;
- Provides derivations of – as well as simple expressions for – the resolution of baseband chromatic dispersion measurements employing both phase- and intensity-modulated optical signals.



# MEASUREMENT OF CHROMATIC DISPERSION USING THE BASEBAND RADIO-FREQUENCY RESPONSE OF A PHASE-MODULATED ANALOG OPTICAL LINK EMPLOYING A REFERENCE FIBER

## II INTRODUCTION

The ability to measure chromatic dispersion is essential for all long-haul communications applications – both analog and digital – as well as for ultrashort pulse photonics. Myriad techniques to measure moderate amounts of chromatic dispersion (i.e., dispersion arising from kilometers of optical fiber and the emphasis of this work) have been demonstrated. Pulse-based time-domain methods such as time-of-flight techniques utilizing a series of pulses at different center wavelengths [1] are widely utilized and various analog / radio-frequency (RF) techniques have been demonstrated as well. The latter include differential phase-shift methods [1, 2] as well as measurement of the baseband (RF) response of an analog optical link employing a lengthy fiber span [3, 4]. The last of these techniques is particularly useful because the measurement apparatus and concept are simple and may be implemented without the need for tunable or short-pulse lasers.

The work presented here is an extension of the work presented in [3, 2] which emphasized the response of a long analog optical link employing intensity modulation (IM) and direct-detection. Here, we extend this work to demonstrate that the phase-response [4] of a long analog link utilizing direct-detection may also be used to characterize the dispersion of a fiber span and, in fact, offers several unique advantages. In addition, we demonstrate a new measurement modality that employs a reference fiber to shift the “zero-dispersion” response of a fiber under test to lower RF frequencies thereby removing one of the primary limitations of the technique presented in [3]. We also show that the measurement resolution is enhanced by utilizing the phase-modulation response over the intensity-modulation response and provide simple expressions for the measurement resolution as a function of the dispersion-length product of the reference fiber and the order and depth of the RF null used to perform the measurement.

This work is organized as follows: in Section III we present a simple analysis illustrating the response of a dispersive medium (described by a quadratic spectral phase variation) to both intensity- and phase-modulated optical carriers. This analysis, which utilizes Fourier theory, is an alternative technique to tracing out the response of an analog link in the time-domain. In Section IV, we present our measurement technique based on the use of optical phase-modulation in concert with a reference fiber; this section also includes the derivation of the measurement resolution and a comparison with that of our technique employing intensity-modulation. In Section V we present measurements that demonstrate our technique. In Section VI we provide a summary of our dispersion measurement technique and results.

## III THE DISPERSION RESPONSE OF A LONG OPTICAL LINK

In this section we derive the response of a dispersive analog optical link to both intensity- and phase-modulated optical signals from a Fourier perspective. Since the bandwidths of the modulation signals in an optical system are significantly less than the optical carrier frequency we may simply work with the slowly-varying complex envelope of the electric field, without the need to carry the optical carrier through explicitly. This simplifies the analysis of our links in that we only need address how our links alter the complex envelope of the modulated optical carrier, not the optical carrier itself and allows use to address the RF response of the system without the tedium of tracing through numerous trigonometric identities.

### Analog Signal Representation under the Slowly-Varying Envelope Approximation

Given the complex envelope  $a(t)$ , we may write the complex electric field of the optical carrier (at frequency  $\omega_o$ ) under the slowly-varying envelope approximation (SVEA) as

$$\tilde{e}(t) = a(t) \exp(j\omega_o t). \quad (1)$$

with the real electric field is then given by

$$e(t) = \text{Real} \{a(t) \exp(j\omega_o t)\}. \quad (2)$$

In the frequency-domain, we may express the optical spectrum in terms of the complex amplitude spectrum of the envelope  $a(t)$

$$A(\omega) = \int_{-\infty}^{\infty} dt a(t) \exp(-j\omega t), \quad (3)$$

which is a complex, baseband function. Because we are working under the SVEA, we will perform our analysis using the complex amplitude spectrum given by Eq. (3). To obtain the real time-domain electric field, we inverse Fourier transform this quantity, multiply by the complex exponential  $\exp(j\omega_o t)$  and take the real part.

From Eq. (1), the time-domain intensity of the optical carrier  $[p(t)]$  averaged over several optical cycles is given by (\* denotes complex conjugation)

$$\begin{aligned} p(t) &= \frac{1}{2} \tilde{e}(t) \tilde{e}^*(t) \\ &= \frac{1}{2} |a(t)|^2. \end{aligned} \quad (4)$$

Note, for the purposes of this work, we may assume the photodiode reproduces the optical intensity faithfully. Therefore, to within a scaling factor (the responsivity of the photodiode)  $p(t)$  as given by Eq. (4) is the measured photocurrent. We may then find the complex amplitude spectrum of the photocurrent via Fourier transform of the intensity as given by Eq. (4).

Now to address the form of the complex envelope  $a(t)$  for amplitude- and phase-modulated optical signals. Here, we are interested in applying a complex modulation signal  $m(t)$  to an optical carrier. In the following, we assume small-signal conditions, i.e. the optical intensity modulation-depth or optical phase-excursion is very small in magnitude. The results are, of course, extensible to cases where this assumption is no longer valid. We express the small signal condition as

$$|m(t)| \ll 1. \quad (5)$$

The complex envelope of the modulated optical carrier is expressed as

$$a(t) \propto 1 + m(t), \quad (6)$$

which yields the optical intensity (photocurrent)

$$p(t) \propto |a(t)|^2 \propto 1 + |m(t)|^2 + 2 \text{Real} [m(t)]. \quad (7)$$

Note, since we assumed a very small modulation-depth [Eq. (5)], the term given by the magnitude-squared of the modulation signal is vanishingly small. Thus, the measured intensity (photocurrent) is proportional to the real part of the modulation signal

$$p(t) \propto 1 + 2 \text{Real} [m(t)]. \quad (8)$$



The complex amplitude spectrum of the intensity (photocurrent) is then

$$P(\omega) \propto \delta(\omega) + 2\mathcal{F}\{\text{Real}[m(t)]\}, \quad (9)$$

where  $\mathcal{F}\{\}$  denotes Fourier transformation.

Under the small-signal approximation, the modulation signal  $m(t)$  is purely real for the case of optical intensity (amplitude) modulation and purely imaginary for optical phase modulation; from Eq. (8) this yields a time-domain optical intensity proportional to the modulation signal  $m(t)$  for an intensity-modulated optical carrier. In the case of a phase-modulated optical carrier, the intensity is constant, as we expect. In the frequency-domain, the complex amplitude spectrum of an intensity-modulated signal is Hermitian, i.e. the real part of the complex amplitude spectrum is an even function of frequency and the imaginary portion is an odd function of frequency. In contrast, the complex amplitude spectrum of a phase-modulated signal is anti-Hermitian; the relations between real- and imaginary parts to even- and odd-functions of frequency are reversed. From Eq. (8) we see the measurable photocurrent variations (RF frequency content) arise from the real part of the modulation envelope  $m(t)$  the Fourier transform of which is a Hermitian function of frequency as given by Eq. (9). Thus, to determine the spectral structure of the photocurrent spectrum, we need only to determine the Hermitian portion of the complex amplitude spectrum of the modulation envelope  $m(t)$ .

### Chromatic Dispersion as a Radio-Frequency Filter

To describe the RF filtering effects of the chromatic dispersion of an optical fiber (intensity-to-phase conversion for IM signals or phase-to-intensity conversion for  $\Phi\text{M}$  signals) we begin with the frequency-dependent propagation constant  $[\beta(\omega)]$  of an electromagnetic wave given by [5]

$$\beta(\omega) = \frac{\omega}{c}n(\omega), \quad (10)$$

where  $c$  is the speed of light in a vacuum and  $n(\omega)$  is the frequency-dependent refractive index. We are interested in the effects of dispersion relative to our carrier frequency – to this end, we can expand the propagation constant in a Taylor series about  $\omega_o$  [6]. Keeping only the quadratic term for the propagation constant yields a quadratic spectral phase which adequately describes the dispersion of optical fiber for reasonable optical bandwidths sufficiently far away from the zero-dispersion wavelength. In terms of frequency offset from the carrier  $\tilde{\omega} = \omega - \omega_o$  this quadratic spectral phase is given by

$$\phi(\tilde{\omega}) = \frac{1}{2}\beta_2 L (\omega - \omega_o)^2, \quad (11)$$

where  $\beta_2$  is the fiber dispersion in  $\text{ps}^2/\text{km}$ . The resulting phase exponential describing propagation through the fiber (assuming a  $+j\omega_o t$  time-dependence for the phasors) is then seen to represent a complex spectral filter function

$$\exp[j\phi(\tilde{\omega})] = \cos\left(\frac{1}{2}\beta_2 L \tilde{\omega}^2\right) + j \sin\left(\frac{1}{2}\beta_2 L \tilde{\omega}^2\right), \quad (12)$$

where we have used Euler's relation to split the exponential into real and imaginary components.

It is important to note that the offset frequency  $\tilde{\omega} = \omega - \omega_o$  corresponds to the frequency content of the modulating signal, that is, baseband radio frequencies. Thus, the filter function in Eq. (12) operates on the complex amplitude spectrum of the optical carrier [Eq. (6)]. The filtered complex amplitude spectrum of the envelope, after passage through a length of optical fiber (length  $L$ , dispersion  $\beta_2$ ), is then

$$A_F(\tilde{\omega}) = A(\tilde{\omega}) \left[ \cos\left(\frac{1}{2}\beta_2 L \tilde{\omega}^2\right) + j \sin\left(\frac{1}{2}\beta_2 L \tilde{\omega}^2\right) \right]. \quad (13)$$

The complex time-domain envelope  $a_F(t)$  is then determined by the inverse Fourier transform of the filtered spectrum. Because we are interested in the RF spectral response of a long-haul analog optical link, we may determine spectral structure of the photocurrent (or RF power) from Eq. (9) directly by selecting the Hermitian portion of the filtered complex amplitude spectrum of the envelope given by Eq. (13).

To determine the difference in RF spectral structure imparted by phase- versus intensity-modulation, we rewrite the complex spectral amplitude of the envelope as

$$A(\tilde{\omega}) = \delta(\tilde{\omega}) + M(\tilde{\omega}). \quad (14)$$

where  $M(\tilde{\omega})$  is the complex amplitude spectrum of the applied modulation envelope  $m(t)$ . Recall, the Hermitian portion of the complex spectral amplitude defined by Eq. (13) gives rise to the RF spectral structure at the link output as shown by Eq. (9). Substituting Eq. (14) into Eq. (13) and recalling that the function  $M(\tilde{\omega})$  is Hermitian for the case of intensity modulation and anti-Hermitian for the case of phase modulation we find that the complex RF spectral amplitude of the photocurrent is given by

$$P_{\text{IM}}(\tilde{\omega}) \propto \delta(\tilde{\omega}) + M(\tilde{\omega}) \cos\left(\frac{1}{2}\beta_2 L \tilde{\omega}^2\right) \quad (15)$$

for the case of intensity modulation and by

$$P_{\Phi\text{M}}(\tilde{\omega}) \propto \delta(\tilde{\omega}) - M(\tilde{\omega}) \sin\left(\frac{1}{2}\beta_2 L \tilde{\omega}^2\right) \quad (16)$$

for the case of phase modulation. The RF power spectrum is then proportional to the magnitude-squared of Eqs. (15) and (16).

#### IV DISPERSION MEASUREMENT USING THE RADIO-FREQUENCY RESPONSE OF AN ANALOG OPTICAL LINK

As previously demonstrated [2, 3] for intensity-modulated analog optical links, the measured RF response may be used to extract the dispersion-length product of the fiber utilized in the link through the location of the nulls in the measured RF link response; as proposed here, the phase modulation response may as well. We see the relationships between the RF nulls and the dispersion-length product for both modulation types by solving for the zeros of Eqs. (15) and (16). For intensity modulation, the zeros of Eq. (15) occur when the argument of the cosine is equal to an odd-multiple of  $\pi/2$  – this yields RF nulls at frequencies given by

$$f_{n,\text{AM}} = \left(\frac{n}{4\pi|\beta_2 L|}\right)^{1/2}, \quad n = 1, 3, 5, \dots, \quad (17)$$

where  $n$  is the null order. For a phase-modulated optical source (in the small-signal regime) the nulls in the RF response occur when the argument of the sine in Eq. (16) equal an integer multiple of  $\pi$ . This condition yields nulls at frequencies given by

$$f_{n,\Phi\text{M}} = \left(\frac{n}{2\pi|\beta_2 L|}\right)^{1/2}, \quad n = 0, 1, 2, \dots \quad (18)$$

Thus, depending on the modulation utilized in an analog link, one may invert either Eq. (17) or Eq. (18) to find the dispersion-length product from the measured location of a null in the RF response.

Examples of the intensity- and phase-modulated responses of 50 km SMF-28 ( $\beta_2 \simeq -22$  ps/nm/km) are shown in Figure 1 (a) and (b), respectively. As is evident, this technique (using an IM or  $\Phi\text{M}$  optical source)

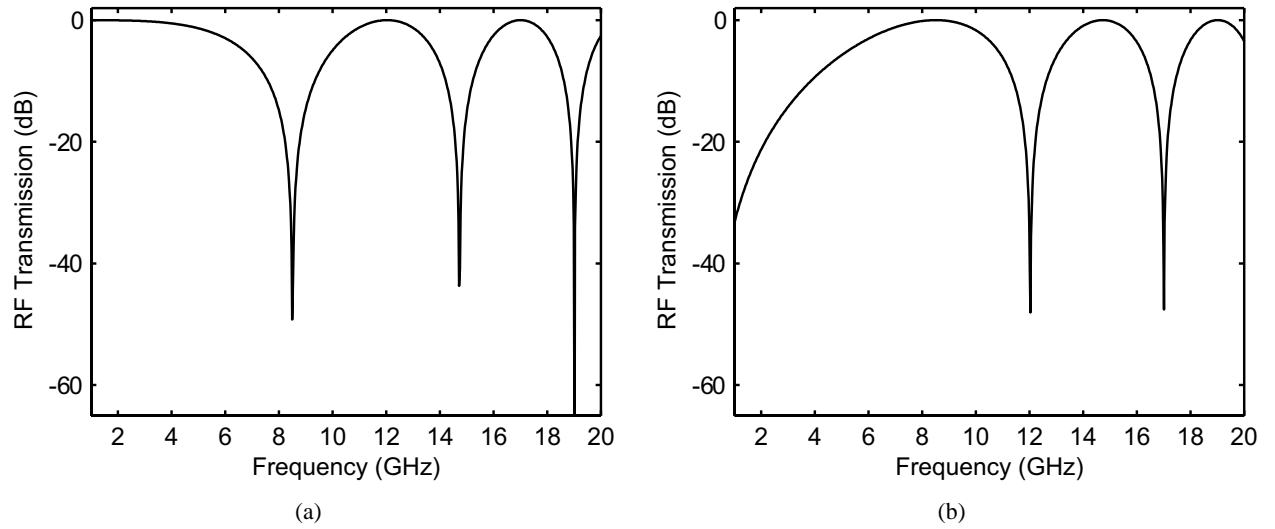


Fig. 1: Calculated RF power response of an analog optical link employing 50 km of Corning SMF-28 single-mode optical fiber ( $\beta_2 \simeq -22 \text{ ps}^2/\text{km}$  at  $\lambda = 1550 \text{ nm}$ ). (a) Intensity modulation response and (b) phase modulation response.

is particularly useful for measuring large dispersion-length products; the first (non-zero order) null moves to lower frequencies as the total fiber dispersion increases. For an amplitude-modulated source, the first null occurs at a frequency of  $f_{n,\text{AM}} \simeq 8.5 \text{ GHz}$  and for a phase-modulated optical source, the first null occurs at approximately  $f_{n,\text{PM}} = 12.0 \text{ GHz}$ . It should be emphasized that this technique only provides the magnitude of the dispersion-length product – it is assumed that the sign of the dispersion is known. To determine the sign of the dispersion has thus far required one to repeat these measurements at a series of optical carrier frequencies (wavelengths) or other techniques entirely, such as the differential phase-shift method [2]. Additionally, to extract the dispersion per-unit-length of the fiber requires a separate measurement of the fiber length [7].

### Measurement of Fiber Dispersion Utilizing a Reference Fiber

As mentioned in the preceding section, the primary limitations of the baseband technique for measuring the dispersion-length product are that the total dispersion must be relatively large to shift the RF nulls to frequencies that are easily measurable (below 20 GHz) and that there is no information provided on the sign of the dispersion. Our technique utilizes a well-characterized reference fiber to shift the “zero-dispersion” response of the test fiber to lower RF frequencies and to provide a reference for the sign of dispersion. In addition, our technique utilizes optical phase modulation which offers the distinct advantage (over intensity modulation which requires a DC modulator bias [8]) that the dispersion measurement does not suffer from modulator bias drift. In IM-based systems, modulator bias drift causes (at a minimum) reduced contrast of the nulls in the RF response. When the modulator bias drifts at a rate faster than the sweep-rate of the RF source utilized in the measurement, broadening of the aforementioned nulls in the RF response also occurs. Both effects lead to reduced accuracy in the measurement of the dispersion-length product.

Our dispersion measurement apparatus is shown schematically in Figure 2. The output of a continuous-wave DFB laser is modulated with a low- $V_\pi$  phase modulator ( $V_\pi \sim 2.9 \text{ V}$ , EOSpace, Inc.) driven by a vector network analyzer (HP 8510, 45 MHz - 50 GHz). The modulated laser then passes through the reference fiber spool (50 km Corning SMF-28) and the fiber under test. The optical heterodyne signal from the

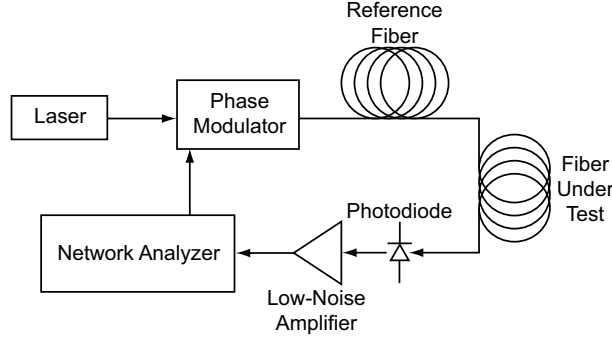


Fig. 2: Baseband dispersion measurement system utilizing phase modulation and a reference fiber.

photodiode (Discovery Semiconductor DSC40S) is amplified with a low-noise amplifier and measured with the network analyzer. The bandwidth of the RF amplifier limits the upper frequency our RF measurements to  $\sim 18$  GHz.

Our measurement utilizes the fact that the RF response depends on the total dispersion of a fiber span. For a span consisting of the reference fiber and the fiber under test, the magnitude of the span dispersion-length-product is given by

$$|\beta_2 L| = |\beta_{2,\text{ref}} L_{\text{ref}} + \beta_{2,\text{u}} L_{\text{u}}|, \quad (19)$$

where the subscripts “ref” and “u” signify the reference and test fibers, respectively. For a phase-modulated optical source and both the reference and test fibers in place, the nulls in the measured RF response are then given by [from Eq. (18) for null  $n$ ]

$$f_{n,\Phi\text{M}} = \left( \frac{n}{2\pi |\beta_{2,\text{ref}} L_{\text{ref}} + \beta_{2,\text{u}} L_{\text{u}}|} \right)^{1/2}. \quad (20)$$

The RF nulls for the reference fiber are given by (determined by measuring the RF response in the absence of the test fiber)

$$f_{n,\Phi\text{M},\text{ref}} = \left( \frac{n}{2\pi |\beta_{2,\text{ref}} L_{\text{ref}}|} \right)^{1/2}. \quad (21)$$

We note, the reference fiber need be characterized only once per measurement session so long as the measurement conditions remain the same (e.g., laser wavelength, temperature). Note, the magnitude of the shift in null location yields information on the magnitude of the dispersion-length product, while the direction of the null shift provides the sign of the dispersion relative to that of the reference fiber. Clearly, when  $f_{n,\Phi\text{M}} < f_{n,\Phi\text{M},\text{ref}}$  the dispersion of the reference and test fibers are of the same sign (the magnitude of the total dispersion increases). If  $f_{n,\Phi\text{M}} > f_{n,\Phi\text{M},\text{ref}}$ , the test and reference fiber dispersions are of opposite sign (the magnitude of the total dispersion decreases). From Eqs. (21) and (20) we obtain the following relations for the dispersion-length product of the test fiber

$$\beta_{2,\text{u}} L_{\text{u}} = \begin{cases} -\text{sgn}(\beta_{2,\text{ref}}) \frac{n}{2\pi} \left| \left( \frac{1}{f_{n,\Phi\text{M},\text{ref}}} \right)^2 - \left( \frac{1}{f_{n,\Phi\text{M}}} \right)^2 \right| & \text{for } f_{n,\Phi\text{M}} > f_{n,\Phi\text{M},\text{ref}} \\ \text{sgn}(\beta_{2,\text{ref}}) \frac{n}{2\pi} \left| \left( \frac{1}{f_{n,\Phi\text{M},\text{ref}}} \right)^2 - \left( \frac{1}{f_{n,\Phi\text{M}}} \right)^2 \right| & \text{for } f_{n,\Phi\text{M}} < f_{n,\Phi\text{M},\text{ref}} \end{cases}. \quad (22)$$

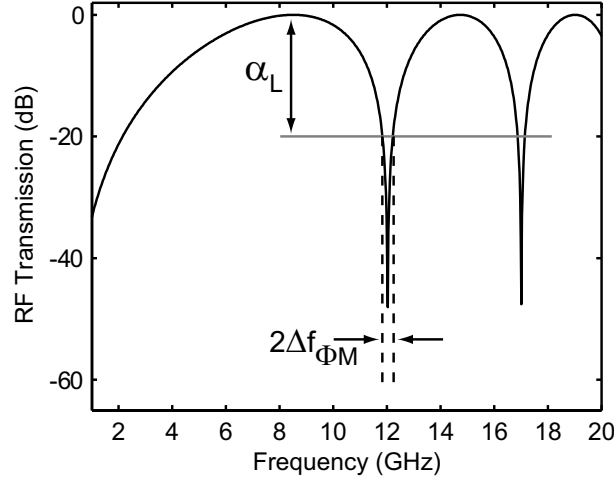


Fig. 3: Graphical definitions of the null-depth  $\alpha_L$  and null half-width (frequency resolution)  $\Delta f_{\Phi M}$  of the phase modulation RF response.

Again, to determine  $\beta_{2,u}$  you must perform an independent measurement of the length of the fiber  $L_u$ . We emphasize that for our technique, the length of the reference fiber is immaterial so long as the desired RF nulls are easily measured; however, it is essential that the (magnitude and sign) of the reference fiber dispersion  $\beta_{2,\text{ref}}$  is well-known. A brief outline of the measurement procedure is given in the Appendix. We note that some prefer to use the dispersion parameter  $D$  instead of  $\beta_2$ ; the relation between these quantities is given by (for  $D$  in ps/nm/km)

$$D = \frac{d\omega}{d\lambda}\beta_2 = -\frac{2\pi c}{\lambda^2}\beta_2. \quad (23)$$

### Measurement Resolution

Applications requiring highly-accurate dispersion characterization require that the resolution of the dispersion measurement technique be well understood. Here, we derive the RF frequency resolution and corresponding dispersion resolution of our technique. The results of this analysis demonstrate that the measurement resolution depends on the total dispersion of the reference fiber, the order of the RF null used to perform the measurement, and the RF measurement fidelity – i.e., the RF null depth. We explicitly derive the resolution for the case of a phase-modulated optical source; the derivation of the resolution for an intensity-modulated source will be briefly outlined at the end of the section. We will also briefly discuss the difference in resolution between the two modulation types and the impact on the overall dispersion measurement.

Since our technique utilizes a relative measurement of dispersion – the shift in frequency of a particular null in the RF response – it is intuitively clear that deeper and narrower RF nulls lead to increased measurement resolution. To see this, we define the null-depth for the reference fiber RF response to be  $\alpha_L$  (dB, relative to the normalized peak RF power response of 0 dB), which yields a linear RF transmission through the link of  $\alpha = 10^{-\alpha_L/10}$ . Figure 3 illustrates the definitions of null-depth and half-width (frequency resolution) for our technique. From the  $\Phi M$  dispersion response [Eq. (16)] we obtain the following expression for the RF transmission  $\alpha$  at the frequency  $\tilde{\omega}_{\alpha_L}$  corresponding to the edge of the RF null at a depth  $\alpha_L$

$$\alpha = \sin^2\left(\frac{1}{2}\phi_{\text{ref}}\tilde{\omega}_{\alpha_L}^2\right), \quad (24)$$

where  $\phi_{\text{ref}}$  corresponds to the dispersion-length product of the reference fiber. Here, the angular frequency  $\tilde{\omega}_{\alpha_L}$  is related to the  $n$ -th null location [Eq. (18)] and the null half-width  $\Delta f_{\Phi M}$  (in Hz, at a null-depth of  $\alpha_L$ ) through the relation

$$\tilde{\omega}_{\alpha_L} = 2\pi (f_{n,\Phi M} \pm \Delta f_{\Phi M}) \quad (25)$$

If we assume reasonable fidelity for the RF measurement (i.e. the null-depth  $-\alpha_L \geq 20$  dB) we may use a first-order Taylor expansion for the sine about the  $n$ -th null. Substituting Eq. (25) into Eq. (24) and solving for the frequency offset ( $\Delta f_{\Phi M}$ ) from the  $n$ -th null location  $f_{n,\Phi M}$  yields

$$\Delta f_{\Phi M} \simeq \frac{1}{2\pi} \frac{\alpha^{1/2}}{\phi_{\text{ref}}} \frac{1}{f_{n,\Phi M}}. \quad (26)$$

Substituting Eq. (18) for  $f_{n,\Phi M}$  into Eq. (26) gives a null half-width (in Hz) of

$$\Delta f_{\Phi M} \simeq \frac{1}{2\pi} \left( \frac{\alpha}{n 2\pi \phi_{\text{ref}}} \right)^{1/2}. \quad (27)$$

Here, we define the frequency resolution of our technique (minimum resolvable frequency shift) to be equal to the half-width  $\Delta f_{\Phi M}$ . As one would expect, the frequency resolution improves with decreasing RF transmission (increasing null depth), null order, and total dispersion of the reference fiber. We determine the fractional dispersion resolution ( $\Delta\phi/\phi_{\text{ref}}$ ) by solving Eq. (22) for RF null locations separated by the frequency resolution  $\Delta f_{\Phi M}$ . Once again assuming reasonable RF null-depth, the fractional dispersion resolution is given by

$$\frac{\Delta\phi}{\phi_{\text{ref}}} \simeq \frac{1}{n} \frac{\alpha^{1/2}}{\pi}. \quad (28)$$

Note, the fractional dispersion resolution depends only on the null order  $n$  and the RF transmission (null depth)  $\alpha$  – not the total dispersion of the reference fiber.

As mentioned previously, intensity modulation may also be utilized in our technique. The frequency- and fractional dispersion resolution for a system using intensity modulation may be derived in a similar manner, beginning from the IM response given in Eq. 15. While we will not present the derivation, the resulting frequency resolution is given (in Hz) by

$$\Delta f_{\text{AM}} \simeq \frac{1}{2\pi} \left( \frac{\alpha}{n\pi\phi_{\text{ref}}} \right)^{1/2}. \quad (29)$$

and the fractional dispersion resolution is

$$\left. \frac{\Delta\phi}{\phi_{\text{ref}}} \right|_{\text{AM}} \simeq \frac{2}{n} \frac{\alpha^{1/2}}{\pi}. \quad (30)$$

It is interesting to note that the frequency resolution for an intensity-modulated system is degraded by a factor of  $\sqrt{2}$  as compared to the  $\Phi M$  system while the fractional dispersion resolution has degraded by a factor of two. The degradation in resolution arises from the fact that, while the RF nulls in the intensity modulation response (for a given order  $n$ ) occur at lower frequencies, they are broader than the corresponding nulls in the  $\Phi M$  response [see Fig. 1 (a) and Eq. (17)]. For dispersion measurements requiring the best possible resolution (i.e., small total dispersion) it is, therefore, advantageous to utilize phase modulation.

To illustrate the capabilities of our technique, the calculated frequency resolution and fractional dispersion resolution (in logarithmic scale) for a 50 km SMF-28 reference fiber are shown in Figure 4 versus the RF null depth and null order. As illustrated by the black curves, the frequency resolution (for the  $n = 1$

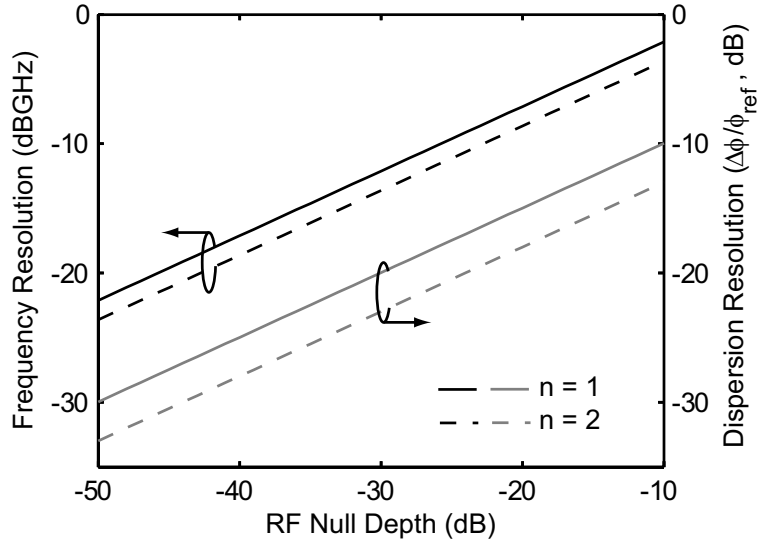


Fig. 4: Calculated frequency resolution (black curves) and fractional dispersion resolution (gray curves) for our measurement system as a function of RF null depth. As expected, both measures of resolution improve with increased null-depth and with null order ( $n = 1$ : solid curves,  $n = 2$ : dashed curves.)

null) ranges from  $\Delta f_{\Phi M} \simeq 500$  MHz for the moderate RF null depth of  $\alpha_L = -10$  dB to  $\Delta f_{\Phi M} \simeq 6$  MHz for nulls on the order of -50 dB in depth. The dashed black curve illustrates the  $n = 2$  null resolution which illustrates the  $1/\sqrt{n}$  dependence on null-order. The gray curves show the fractional dispersion resolution as a function of null depth for the  $n = 1$  (solid) and  $n = 2$  (dashed) nulls. Note the fractional dispersion resolution improves as  $1/n$ ; here, this leads to a 3-dB improvement in resolution. For shallow nulls ( $\alpha_L \simeq -10$  dB, the  $n = 1$  fractional dispersion resolution is on the order of  $\Delta\phi/\phi_{\text{ref}} \simeq -10$  dB (10%) (for  $n = 2$ , -13 dB or 5%). For very deep nulls, on the order of  $\alpha_L = -50$  dB, the first-order fractional dispersion resolution can reach approximately  $\Delta\phi/\phi_{\text{ref}} = -30$  dB or 0.1% ( $n = 2$ , 0.05%). It is important to note that, while a null depth of -50 dB is readily achieved for many fibers, the limits of fiber attenuation, stimulated Brillouin scattering (which limits the optical launch power), and the RF system response may limit the null depth to as little as  $\alpha_L = -30$  – -20 dB. The actual net dispersion resolution depends on the total dispersion of the reference fiber. As an example, a reference fiber consisting of a 50 km length of single-mode fiber ( $\phi_{\text{ref}} = \sim 22 \text{ ps}^2/\text{km} \times 50 \simeq 1100 \text{ ps}^2$ ) and a first-order null depth of  $\alpha_L = -30$  dB yields a net dispersion resolution of approximately  $\Delta\phi = 11 \text{ ps}^2$  (1% of the total dispersion.)

## V EXPERIMENTAL RESULTS

We now present several examples of our measurement technique [see Fig. 3 for the experimental apparatus]. We begin by characterizing the RF response (dispersion) of our 50-km SMF-28 reference fiber. After calibrating the network analyzer to the photodiode and RF amplifier responses, we measure the RF transmission shown by the black curve in Figure 3 (a). The locations of the first two (non-zero order) RF nulls are found to be  $f_1 \simeq 12.23$  GHz and  $f_2 \simeq 17.22$  GHz. Averaging the  $\beta_2$  values computed from Eq. (21) (after inverting to solve for the magnitude of the dispersion-length product) for a  $L_{\text{ref}} = 50$  km reference fiber we obtain a reference fiber dispersion of  $\beta_2 = -21.37 \text{ ps}^2/\text{km}$  (note, our measurement yields the magnitude of  $\beta_2$ , the sign is known to be negative for SMF-28); for a wavelength of  $\lambda = 1551.27$  nm, this yields a dispersion pa-

parameter of  $D_{\text{meas}} = 16.74$  ps/nm/km which shows excellent agreement with that calculated from information provided by Corning ( $D_{\text{calc}} = 16.23$  ps/nm/km). To characterize the fidelity of our measurement apparatus, we measure the RF null-depth (for both the first- and second-order nulls to be  $\alpha_L \simeq -50$  dB; this yields a fractional dispersion resolution (for the  $n = 1$  RF null) of  $\Delta\phi/\phi_{\text{ref}} \simeq -30$  dB (0.1%). The achievable net dispersion resolution is then approximately  $\phi = 1.1$  ps<sup>2</sup>. The measured null-width (frequency resolution) shows excellent agreement with that predicted by Eq. (27). For the  $n = 1$  null, the measured (-30 dB) width is  $\Delta f_{\Phi M} = 59.5$  MHz – extremely close to the theoretical value of 61.4 MHz. For the second-order null, the measured -30 dB width of  $\sim 35$  MHz again agrees quite well with the calculated value of 43 MHz (the 8 MHz difference is below the 24 MHz VNA data step).

After determining the position of the first RF null in the response of our reference fiber, we place a  $L_u = 25$  km length of Corning LEAF non-zero dispersion-shifted optical fiber into our measurement apparatus. As the dispersion of LEAF fiber is of the same sign as that of SMF-28, we expect the nulls in the RF response to shift to lower frequencies as the magnitude of the total dispersion of the fiber span (reference + test fiber) increases. As illustrated by the gray curve in Fig. 3 (b), this is indeed what occurs. The measured null locations [given by Eq. (20)] are found to be  $f_{n,\Phi M,1} \simeq 11.55$  GHz and  $f_{n,\Phi M,2} \simeq 16.27$  GHz. Using the first measured RF null and a fiber length of  $L_u = 25$  km in Eq. (22) we obtain a dispersion of  $\beta_2 = -5.16$  ps<sup>2</sup>/km yielding a dispersion parameter of  $D_{\text{meas}} = 4.04$  ps/nm/km. The calculated dispersion parameter agrees quite well with the  $D_{\text{calc}} = 4.43$  ps/nm/km value calculated from a first-order Taylor expansion based on the dispersion information provided by Corning.

To illustrate the measurement for a fiber exhibiting dispersion opposite to that of the reference fiber, we replace the length of LEAF fiber with a  $L_u = 17.5$  km length of Corning Metrocor fiber (calculated dispersion parameter of  $D_{\text{calc}} = -7.45$  ps/nm/km at 1551.27 nm). The measured RF response of the fiber span (reference + Metrocor) is shown by the gray curve in Figure f3 (b). As expected, the measured RF nulls shift to higher frequencies as compared to those of the reference fiber (black curve) as the total dispersion of the fiber span has decreased. From the position of the first RF null  $f_{\Phi M,1} \simeq 13.14$  GHz, we calculate a dispersion of  $\beta_2 = 8.13$  ps<sup>2</sup>/km; the measured dispersion parameter is then  $D_{\text{meas}} = -6.37$  ps/nm/km which shows reasonable agreement with the specified value above. We should emphasize that the calculated dispersion parameter values are included as a reference only and should not be interpreted as “known” values. Dispersion is found to vary significantly from one spool of fiber to the next; in this work, the differences in measured versus calculated values are well within these variations.

## VI SUMMARY

We present and demonstrate a new chromatic dispersion measurement technique based on the baseband radio-frequency response of a phase-modulated analog optical link. Our technique utilizes a well-characterized reference fiber (i.e. known dispersion) to shift the “zero-dispersion” response of the baseband technique to moderate RF frequencies ( $\sim 10$ – $12$  GHz). Additionally, incorporation of the reference fiber enables our technique to determine both the magnitude and sign of the dispersion of a test fiber – a significant improvement over previously demonstrated baseband techniques. The use of phase modulation (as opposed to intensity modulation) improves the measurement stability and fidelity by removing the effects of modulator bias drift. This technique is capable of providing highly accurate dispersion measurements ( $\leq 1\%$  of the net dispersion of the reference fiber) for fiber lengths ranging from  $\sim 100$  m to  $100+$  km.



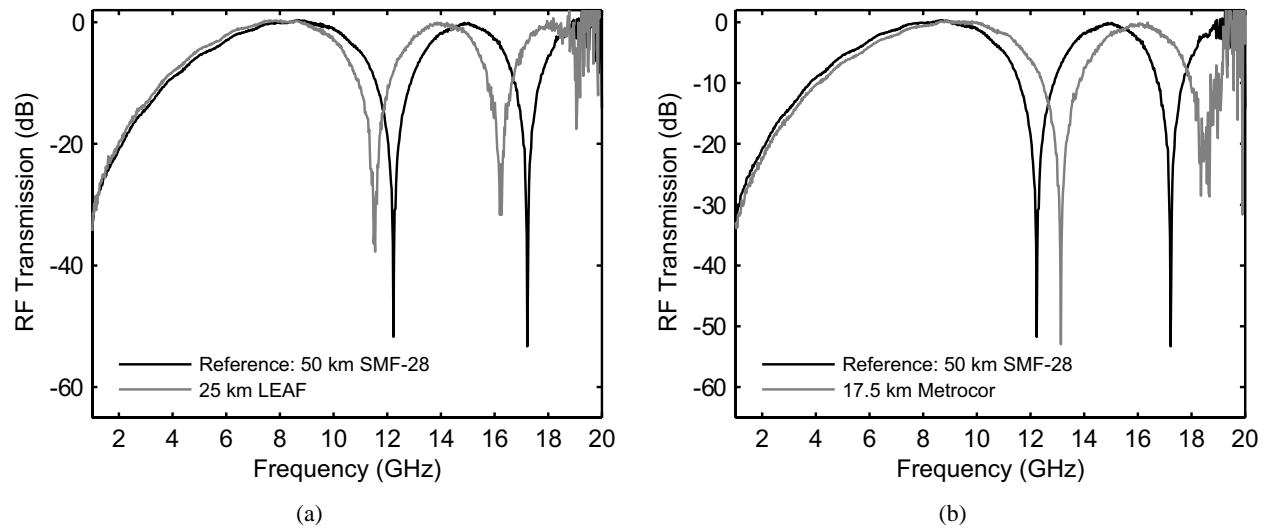


Fig. 5: Measured RF responses for the 50-km SMF-28 reference fiber [black curve in both (a) and (b)], (a) 25 km LEAF non-zero dispersion-shifted fiber, and (b) 17.5 km Metrocor dispersion compensating fiber.



## APPENDIX A

### VII APPENDIX-A – OUTLINE OF THE MEASUREMENT TECHNIQUE

This section is intended to give a brief outline of how to perform a dispersion measurement using our technique. We direct the reader to Figure 2 for a schematic of the measurement apparatus. It is assumed that the network analyzer has been connected to the phase modulator and RF amplifier as shown in Figure 2 and that the reader is familiar with network analyzer measurements.

- *Characterize the Reference Fiber*

1. Measure the RF response of the optical link constructed with only the reference fiber.
2. Measure the location of the first (or first several) nulls in the RF response. These correspond to the frequencies  $f_{n,\Phi_{M,\text{ref}}}$  in Eq. (22). Note,  $n$  represents the null order. The RF null frequencies are related to the fiber dispersion-length product through Eq. (18).

- *Characterize the Test Fiber*

1. Insert the test fiber into the optical link.
2. Measure the RF response of the optical link consisting of both the reference and test fibers.
3. Measure the location of the first (or first several) nulls in the RF response. These correspond to the frequencies  $f_{n,\Phi_M}$  in Eq. (22). Note,  $n$  again represents the null order.
4. Based on the direction of null shift when including the test fiber, use the appropriate sign of dispersion (relative to that of the reference fiber) to calculate the dispersion-length product ( $\beta_{2,u}L_u$ ) of the test fiber [see Eq. (22) and the associated discussion].

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